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Algorithmic Aspects of Data Analytics and Machine Learning

SS 2025 — Sheet 9

https://aam.uni-freiburg.de/agba/lehre/ss25/algml/index.html

Due: July 4, 2025, 2 p.m.

Task 1

Let $A, B \subset \mathbb{R}^2$ be nonempty, convex, and compact sets with

 $A\cap B=\emptyset.$

Show that there exists an affine hyperplane that separates A and B, i.e. there exist a vector $w \in \mathbb{R}^2$ with $w \neq 0$ and a scalar $c \in \mathbb{R}$ such that:

 $\forall a \in A : \langle w, a \rangle < c \quad \text{and} \quad \forall b \in B : \langle w, b \rangle > c.$

Task 2

Suppose the perceptron algorithm outputs the combined weight vector

$$w = (2, 1, 1).$$

Sketch the decision boundary in \mathbb{R}^2 and indicate which points are classified as -1 and which as +1.

Task 3

The classical Perceptron algorithm is designed for binary classification with labels in $\{-1, +1\}$. Suppose we instead work with labels in $\{0, 1\}$.

- (i) Explain why the update condition $y_i \langle w, \tilde{x}_i \rangle \leq 0$, used in the standard Perceptron algorithm, does not work for labels in $\{0, 1\}$.
- (ii) Modify the update rule that allows the Perceptron algorithm to work with labels in $\{0, 1\}$.

Task 4

(4 Points)

x_1	x_2	$AND(x_1, x_2)$		x_1	x_2	$XOR(x_1, x_2)$
F	F	F	ĺ	F	F	F
F	T	F		\mathbf{F}	Т	Т
Т	F	F		Т	F	Т
Т	Т	Т		Т	Т	F

(a) Replace "T" (true) with +1 and "F" (false) with -1 in the truth table for the **AND** function. Then run the perceptron algorithm manually on this training set, starting with the combined weight vector

$$w^{(0)} = (1, 2, 3).$$

(b) Repeat the same process for the **XOR** function.

(4 Points)

(4 Points)

(4 Points)