Exercises
Mathematical Modeling

Sheet 2

Due: Wednesday 21.05.2025, 14:00.
Letterbox 3.21 in the basement of Ernst-Zermelo-Str.1

Please hand in as pairs of students

Exercise 5: (4 Points)

Consider a direct current network with incidence matrix B, conductance matrix C, and the vectors x, y, b for the potentials, the currents and the voltage sources.

(a) It is well known that the power dissipated at the resistor is $P = UI$, where U is the voltage drop and I the current. Find a representation for the *total* power dissipated in the network.

(b) The power provided by a voltage source is again given by $P = UI$, where U is a supplied voltage and I is the withdrawn current. Find a representation of the power provided by all voltage sources.

(c) Show that the quantities from (a) and (b) are identical.

Exercise 6: (4 Points)

Consider the system of equations $Mz = f$, where $M = \begin{pmatrix} A & B \\ B^T & 0 \end{pmatrix}$, $A \in \mathbb{R}^{n \times n}$, $B \in \mathbb{R}^{n \times m}$, and $f \in \mathbb{R}^{n+m}$. We suppose that A is symmetric and positive semidefinite.

(a) Show: $y \in \text{Ker}(A) \Leftrightarrow y^T Ay = 0$

(b) Compute the kernel of M depending on the kernels of A, B and B^T.

(c) Characterize the right-hand side vectors f, for which the system of equation $Mz = f$ has a solution.

Exercise 7: (4 Points)

Let $C \in \mathbb{R}^{n \times n}$ be symmetric and regular, but not necessarily positive definite, and $B \in \mathbb{R}^{n \times m}$. For $M = B^T CB$, does the assertion $\text{Ker}(M) = \text{Ker}(B)$ hold? If so, give a proof, otherwise give a counterexample.

Exercise 8: (4 Points)

Formulate a system of equations for the computation of the voltages and currents for the alternating current circuit on the next page, with angular frequency $\omega = 50/s$:
Use the following voltage sources:

(a) $U_1(t) = 2\,V \cos(\omega t), U_2(t) = 2\,V \cos(\omega t)$

(b) $U_1(t) = 8\,V \cos(\omega t), U_2(t) = 8\,V \sin(\omega t)$

(c) $U_1(t) = 8\,V \cos(\omega t), U_2(t) = 8\sqrt{2}\,V \cos(\omega t - \frac{\pi}{4})$

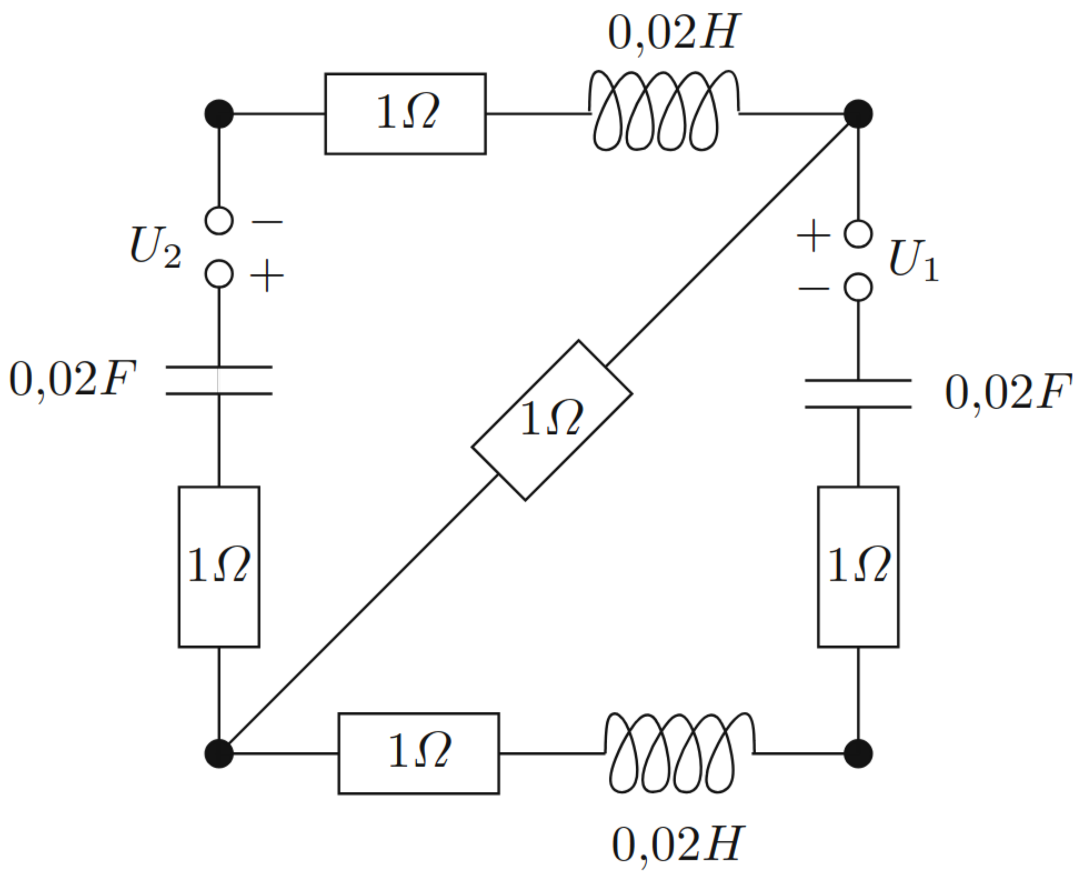

Figure 1: Alternating current circuit from Exercise 8.