Exercises
Mathematical Modeling

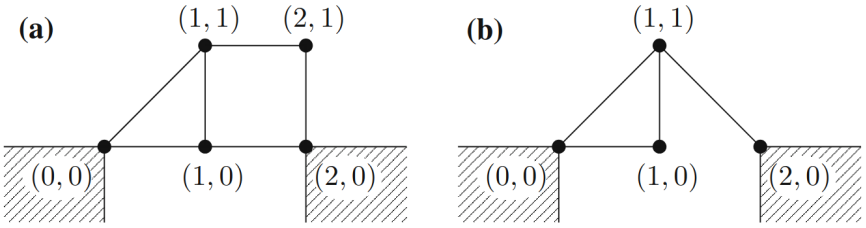
Sheet 3

Due: Wednesday 04.06.2025, 14:00.
Letterbox 3.21 in the basement of Ernst-Zermelo-Str.1

Please hand in as pairs of students

Exercise 9: (2+2 Points)

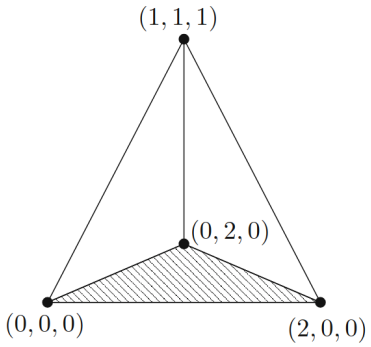
Decide whether the space frames depicted are statically determinate, statically undeterminate or unstable:



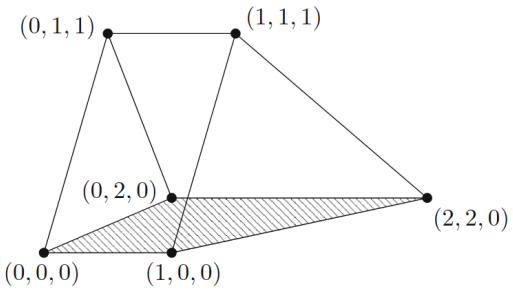
Exercise 10: (2+2 Points)

In the following three-dimensional space frame all moduli of elasticity are supposed to be 1.

(i) Compute the distribution of stresses in the following space frame with fixed nodes $(0,0,0)$, $(2,0,0)$, $(0,2,0)$ and a force $f = (\frac{\sqrt{3}}{50}, 0, 0)$, acting at node $(1,1,1)$:



(ii) Show that the following space frame with fixed nodes with $(0,0,0), (1,0,0), (0,2,0)$ and $(2,2,0)$ is unstable. What conditions must be met by the force vectors acting at the nodes $(0,1,1)$ and $(1,1,1)$, such that the resulting linear system of equations nevertheless has a solution?



Exercise 11: (2+1+1 Points)

(i) Show by the formal equivalence

$$\frac{dy}{dt} = f(t)g(y) \iff \frac{1}{g(y)}\,dy = f(t)\,dt \iff \int \frac{1}{g(y)}\,dy = \int f(t)\,dt,$$

that the function $y(t) = G^{-1}(F(t)+c)$, where $G(y)$ is an antiderivative of $1/g(y)$ and $F(t)+c$ is an antiderivative of $f(t)$, solves the differential equation $y' = f(t)g(y)$. Discuss sufficient conditions for the well-posedness of this representation.

(ii) How can initial conditions be incorporated, and to what extent does uniqueness of the solution hold?

(iii) Construct a nontrivial solution for the initial value problem $y' = y^{2/3}$, $y(0) = 0$.

Exercise 12: (4 Points)

Let $(y_l)_{l=0,\dots,K}$ be a nonnegative sequence and let $\alpha, \beta \geq 0$ such that for $l = 0, 1, \dots, K$ the estimate

$$y_l \leq \alpha + \sum_{k=0}^{l-1} \beta y_k$$

holds. Show that $y_l \leq \alpha(1+\beta)^l \leq \alpha \exp(K\beta)$ for $l = 0, 1, \dots, K$.
This shows the discrete version of Gronwall's lemma if we write $\beta = \tilde{\beta}\tau$ with timestep τ.