

Exercises Mathematical Modeling

Sheet 5

Due: Wednesday 09.07.2025, 14:00.
Letterbox 3.21 in the basement of Ernst-Zermelo-Str.1

Please hand in as pairs of students

Exercise 17:

(3=1+1+1 Points)

Which quadrature formulas underlie the classical Runge–Kutta method, the 3/8-rule, and the Radau-3 method with three nodes? What are their respective orders of accuracy?

Exercise 18:

(4 Points)

Construct a Runge–Kutta method of consistency order $p = 4$ based on Simpson’s rule.

Exercise 19:

(4 Points)

Let $A \in \mathbb{R}^{n \times n}$ be negative definite, i.e., there exists $\alpha > 0$ such that

$$z^\top A z \leq -\alpha \|z\|^2 \quad \text{for all } z \in \mathbb{R}^n.$$

Show that the solution of the initial value problem

$$\dot{y} = Ay, \quad y(0) = y_0$$

converges exponentially fast to zero for any initial value $y_0 \in \mathbb{R}^n$.

Exercise 20:

(4=2+2 Points)

Consider the Runge–Kutta method defined by the Butcher tableau:

$$\begin{array}{c|cc} \frac{1}{3} & \frac{5}{12} & -\frac{1}{12} \\ 1 & \frac{3}{4} & \frac{1}{4} \\ \hline & \frac{3}{4} & \frac{1}{4} \end{array}$$

Investigate whether this method is A-stable and/or L-stable.