Programming Exercises
Mathematical Modeling

Sheet 3

Due: Wednesday 11.06.2025, 14:00,
Per email at *eric.trebuchon@math.uni-freiburg.de*
Please write your programm in **Octave** or **Python**

Please hand in as pairs of students

Exercise 4: (16=3+3+3+3+4 Points)

In Python, differential equations can be approximately solved using the routine `solve_ivp` in scipy.integrate. The routine `solve_ivp` delivers a list `t_vec` of time points

$$0 = t_0 < t_1 < \cdots < t_N = T$$

and a matrix `y_vec` containing the corresponding approximations $\tilde{y}(t_i)$ to the exact solution values $y(t_i)$ at the times t_i, for $i = 0, 1, \ldots, N$. Use this (or other routines) to approximately solve the following initial value problems and to plot the approximate solutions:

(i) The initial value problem for the predator-prey model

$$y_1' = \alpha\, y_1(1 - y_2),$$
$$y_2' = \beta\, y_2(y_1 - 1)$$

on the interval $[0, T]$ with $T = 10$, $\alpha = 2$, $\beta = 1$, and initial conditions $y_1(0) = 3$, $y_2(0) = 1$.

(ii) The initial value problem for the spring-mass oscillator

$$my'' + ry' + D(y - \ell) = 0$$

on the interval $[0, T]$ with $T = 1$, $m = 1$, $D = 1$, $\ell = 1$, and various values $r \in \{0, 1, 5\}$, with initial conditions $y(0) = \ell$, $y'(0) = 1$.

(iii) The initial value problem for the undamped pendulum

$$y'' = -\frac{g}{\ell}\sin(y)$$

with $g = 1$, $\ell = 1$, and initial conditions $y(0) = 0$, $y'(0) \in \{1, 2, 4, 8\}$.

(iv) The initial value problem

$$y'' - Ny' - (N + 1)y = 0$$

on the interval $[0, 1]$ with initial conditions $y(0) = 1$, $y'(0) = -1$, whose exact solution is given by $y(t) = \mathrm{e}^{-t}$, for $N = 1, 2, 10$ and for small perturbations of the initial condition $y(0) = 1$.

(v) Solve and plot the two-body problem

$$m_1 y_1'' = \gamma \frac{m_1 m_2}{\|y_1 - y_2\|^2} \frac{y_2 - y_1}{\|y_1 - y_2\|}, \qquad m_2 y_2'' = \gamma \frac{m_1 m_2}{\|y_1 - y_2\|^2} \frac{y_1 - y_2}{\|y_1 - y_2\|}$$

for various initial data and mass ratios $m_1/m_2 \in \{1, 2, 10\}$.

Construct initial data both that give rise to solutions well-defined for all positive times, and initial data for which the solution exists only on a finite time interval.