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# Theorie und Numerik partieller Differentialgleichungen: Blatt 9

**Abgabe**: Abgabe Mittwoch, 25.01.23 vor der Vorlesung.

**Aufgabe 1** (4 Points)**.** *Contractivity for Gradient Flows.*

**Theorem**: Assume that f is $\lambda$-convex for some $\lambda \in \mathbb{R}$ and lower semi-continuous. Let $x(t) = S_t\bar{x}$ be a gradient flow starting from $\bar{x} \in \overline{\mathrm{Dom}(f)}$. Prove the contractivity property

$$|S_t\bar{x} \circ S_t\bar{y}| \leq e^{-\lambda t}|\bar{x} - \bar{y}|, \quad \forall \bar{x}, \bar{y} \in \overline{\mathrm{Dom}(f)}$$

in the finite dimensional case for arbitrary $\lambda \in \mathbb{R}$.

**Aufgabe 2** (4 Points)**.** *Contractivity for EVI$_\lambda$-solutions.*

Prove the contractivity statement for EVI$_\lambda$-solutions in the finite dimensional case for arbitrary $\lambda \in \mathbb{R}$.

**Aufgabe 3** (4 Points)**.** *Finite Differences.*

Prove that

$$\begin{aligned}\lim_{h\to 0^+} \frac{\delta(t+h,t+h) - \delta(t,t)}{h} \leq & \limsup_{h\to 0^+} \frac{\delta(t+h,t) - \delta(t,t)}{h} \\ & + \limsup_{h\to 0^+} \frac{\delta(t,t+h) - \delta(t,t)}{h},\end{aligned}$$

for almost all $t$, $\delta(s,t) = \frac{1}{2}|x(t) - \tilde{x}(t)|^2$ and $AC$-curves $x$, $\tilde{x}$.

**Aufgabe 4** (4 Points)**.** *Descending slope.*

Compute the descending slope

$$\left|\nabla^- f\right|(u) = \limsup_{v\to u} \frac{(f(v) - f(u))^-}{|v-u|}$$

for the following functionals (defined on $L^2$, set to $+\infty$ wherever not defined).

(i) $f(u) = \int_\Omega |\nabla u|^2 \,\mathrm{d}x$,
(ii) $f(u) = \int_\Omega |\nabla u| \,\mathrm{d}x$,
(iii) $f(u) = \int_\Omega |\nabla u|^2 + u^2(1-u)^2 \,\mathrm{d}x$.