University of Freiburg
Department for Applied Mathematics
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Winter Term 2025

Introduction to the Theory and Numerics for Partial Differential Equations
Computer Practical

Series 4
Return: November 26, 2025
Computer Wave

Project 5 (16 Points). *Schemes for the Wave Equation*

(1) Numerically solve the wave equation $u_{tt} - u_{xx} = 0$ in $(0,1) \times (0,T)$ with homogeneous Dirichlet boundary conditions $u(x,t) = 0$ for $x \in \{0,1\}$, $0 \le t \le T$ and initial conditions $u_t(x,0) = h(x) = 0$ and $u(x,0) = g(x) = \sin(\pi x)$, using an explicit difference scheme with discretization parameters

$$(\tau, h) = \frac{1}{40}(2,2), \quad (\tau, h) = \frac{1}{40}(2,1), \quad (\tau, h) = \frac{1}{40}(1,2).$$

Compare your results and explain differences in the numerical solutions.

(2) Change the initial conditions to

$$u(x,0) = 0, \quad u_t(x,0) = \begin{cases} 1 \text{ if } 0.4 \le x \le 0.6, \\ 0 \text{ otherwise }, \end{cases}$$

and run the program for different pairs of discretization parameters.

(3) Experimentally investigate the violation of a discrete version of the energy conservation principle by plotting the quantity

$$\Gamma^k = \frac{\tau}{2} \sum_{j=1}^{J-1} \left| \partial_t^+ U_j^k \right|^2 + \frac{h}{2} \sum_{j=1}^{J} \left| \partial_x^- U_j^k \right|^2$$

as functions of $k = 0, 1, \ldots, K-1$.

(4) Expand your implementation to the spatial domains $(0,1)^2$ (using the 5-point Laplace from Poisson's equation on the square) and $(0,1)^3$ (using the analogous finite difference Laplacian in three dimensions). Use a very localized disturbance at the center of the spatial domain as initial condition and investigate Huygen's principle.