Abteilung für
Angewandte Mathematik

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Numerik für Differentialgleichungen – SoSe 2023

Sheet 1

Ausgabe: 26.04.2023, 12:00 Uhr   Abgabe: 03.05.2023, 12:00 Uhr

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Homepage to the lecture:

`https://aam.uni-freiburg.de/agsa/lehre/ss23/ndgln`

Exercise 1 (1+1+2 points)**.** In this exercise we want to explore *separation of variables.* Consider the ODE

$$y' = f(t)g(y). \tag{1}$$

Let G and F be antiderivatives of $\frac{1}{g}$ and f respectively and let all functions involved be continuous.

(a) We assume that all involved functions exist and are sufficiently smooth. Prove that $y(t) = G^{-1}(F(t) + c)$ solves (1) for any $c \in \mathbb{R}$.

(b) What requirements do we have to place on g for this method to work? Why does G^{-1} exist?

(c) Solve the following initial value problem using (a):

$$y' = y^2 \frac{1}{1+t}, \quad y(0) = 1.$$

Can we choose $(-1, \infty)$ as an existence interval for y?

Exercise 2 (2+2 points)**.** Consider the mathematical pendulum of length l, to the end of which a weight of mass m is attached, see Fig. 1. Here ϕ denotes the angle between the vertical axis and the pendulum.

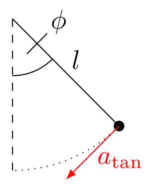

FIG. 1. The mathematical pendulum

(a) Derive a formula for the tangential acceleration $a_{\tan}$ to show that $\phi : [0, T] \longrightarrow \mathbb{R}$ satisfies the differential equation $\phi'' = -(g/l)\sin(\phi)$, where g is the gravitational constant. You don't need to consider frictional effects.
Hint: $F_{tan} = m \cdot a_{tan}$.

(b) Simplify the differential equation for small angles ϕ and solve the resulting differential equation with respect to $\phi(0) = \phi_0$ and $\phi'(0) = \omega_0$.

Exercise 3 (1+1+2 points)**.** Consider the differential equations $y' = ty$, $y' = \sin(t)y$ and $y' = \cos(t)e^y$ for which $y(0) = y_0$. Compute all non-trivial solutions with respect to $y(0) = y_0$ and specify the maximal value for T, such that the solution exists for all $t \in [0, T)$.

Exercise 4 (4 points)**.** Draw the phase diagram for the following system of differential equations

$$y_1' = y_1(1 - y_2), \quad y_2' = y_2(y_1 - 1)$$

on the square $[0,5]^2$ and explain the occurance of periodic solutions.
Hint: First eliminate the time variable to get an equation including dy_1 *and* dy_2*. Integrate this equation to get the implicit relation* $F(y_1, y_2) = c$ *for some function* F *and some constant* c*.*