Abteilung für
Angewandte Mathematik

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Numerik für Differentialgleichungen – SoSe 2023

Sheet 4

Ausgabe: 14.06.2023, 12:00 Uhr   Abgabe: 21.06.2023, 12:00 Uhr

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Homepage to the lecture:

`https://aam.uni-freiburg.de/agsa/lehre/ss23/ndgln`

Exercise 1 (4 points)**.** Which quadrature formulas are the basis of the *classical Runge-Kutta method*, the *3/8-rule* and *the Radau-3 method* and which degrees of exactness do they have?

Exercise 2 (4 points)**.** Determine the Butcher-Tableau for the Runge-Kutta method defined by the following increment function:

$$\begin{aligned}\phi(t, y, \tau) &= \frac{1}{6}(\eta_1 + 4\eta_2 + \eta_3),\\ \eta_1 &= f(t, y),\\ \eta_2 &= f(t + \tau/2, y + \tau\eta_1/2),\\ \eta_3 &= f(t + \tau, y + \tau(-\eta_1 + 2\eta_2)).\end{aligned}$$

Prove that this method has consistency order $p = 3$.

Exercise 3 (4 points)**.** Derive sufficient conditions for the third order consistency of a *Runge-Kutta method* for the case of autonomous differential equations.
Hint: Use the derivation of a sufficient condition for the second order consistency from Remark 22.5 in the book Numerics 3x9.

Exercise 4 (1 + 3 points)**.** Let $A \in \mathbb{R}^{m\times m}$ such that $||A|| < 1$ with respect to an operator norm $||\cdot||$.

(i) Prove that $(E_m - A)$ is invertible with inverse $(E_m - A)^{-1} = \sum_{n=0}^{\infty} A^n$.
(ii) Formulate *Newton's method* for solving the fixed point equation $\eta = \Phi(\eta)$ for determining a coefficient vector $\eta \in \mathbb{R}$ in a *Runge-Kutta method* and discuss its well-posedness.