Abteilung für
Angewandte Mathematik

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Numerik für Differentialgleichungen – SoSe 2023

Sheet 5

Ausgabe: 28.06.2023, 12:00 Uhr — Abgabe: 12.07.2023, 12:00 Uhr

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Homepage to the lecture:

https://aam.uni-freiburg.de/agsa/lehre/ss23/ndgln

Remark: This is the last theory sheet. As you can see, there are $4 + 3$ exercises, so you can achieve 12 extra points in case you need some of them.

Exercise 1 (4 points)**.** Compute the order of the *leap-frog* method

(1) by considering

$$y'(t_k) = \frac{y(t_{k+1}) - y(t_{k-1})}{2\tau},$$

(2) by checking the general consistency criterion for linear multistep methods.

Exercise 2 (4 points)**.** Construct a multistep procedure by expressing the integral for

$$y(t_{k+2}) = y(t_k) + \int_{t_k}^{t_{k+2}} f(s, y(s))\,\mathrm{d}s$$

using Simpson's rule, and determine the consistency order of the resulting procedure.

Exercise 3 (4 points)**.** Show that the Adams-Moulton method is well-defined under the condition $\tau||\beta||_1 L < 1$, where L is the uniform Lipschitz constant of the function f belonging to the differential equation.

Exercise 4 (4 points)**.** Let $f \in C^1([0, T] \times \mathbb{R})$ with $|\partial_z f(t, z)| \leq C$ for all $(t, z) \in [0, T] \times \mathbb{R}$. Prove that the *Adams-Moulton, Adams-Bashforth,* and *Adams-Bashforth-Moulton methods* satisfy the conditions of the general convergence statement for multi-step methods

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Extra points

Exercise 5 (4 points)**.** Examine the zero stability of the following recursive sequences:

a) The *Fibonacci-sequence*: $y_{k+2} = y_{k+1} + y_k$.
b) The *Tschebyscheff-recursion*: $T_{k+2}(x) = 2xT_{k+1}(x) - T_k(x)$.

Exercise 6 (4 points)**.** Let $g : \mathbb{R}^n \longrightarrow \mathbb{R}$ be continuously differentiable and let $f : \mathbb{R}^n \longrightarrow \mathbb{R}^n$ be defined by $f = -\nabla g$. Prove that every solution $y : [0, T] \longrightarrow \mathbb{R}$ of the initial value problem $y' = f(y), y(0) = y_0$ satisfies

$$\int_0^t |y'(s)|^2\,\mathrm{d}s + g(y(t)) = g(y_0).$$

Exercise 7 (4 points)**.** Use the explicit and implicit Euler methods to compute approximated solutions at time $T = 1$ for the initial value problem $y' = 2\alpha t y, y(0) = 1$. Use step sizes $\tau = 1/2^l, l = 1, 2, 3$ and $\alpha = \pm 3$. Compare the approximated solutions with the exact solution.