Abteilung für
Angewandte Mathematik

JProf. Dr. Diyora Salimova
M.Sc. Mario Keller

Praktikum zur Vorlesung: Numerik für Differentialgleichungen – SoSe 2023

Sheet 1

Ausgabe: 03.05.2023, 12:00 Uhr — Abgabe: 10.05.2023, 12:00 Uhr

Homepage to the lecture:

https://aam.uni-freiburg.de/agsa/lehre/ss23/ndgln

Project 1 (8 points)**.** In MATLAB, differential equations can be solved approximatively with the routine `ode45`. In the case of the system $y' = f(t,y)$ in the interval $[0,T]$ with initial condition $y(0) = y_0$ this is realized for the mapping $f(t,y) = Ay$ in the MATLAB-program `test_ode` which you can download from the lecture homepage. The routine `ode45` returns a list `t_vec` of time points $0 = t_0 < t_1 < \ldots < t_N = T$ and a matrix `y_vec` with corresponding approximations $\tilde{y}(t_i)$ of the exact solution values $y(t_i)$ at the times t_i for $i = 0, 1, \ldots, N$. Modify the program `test_ode.m` to solve the following initial value problems approximatively and to display the approximation solutions graphically:

(i) The *Lotka-Volterra* equations (see Sheet 1 from theory)

$$y_1' = y_1(1 - y_2), \quad y_2' = y_2(y_1 - 1)$$

on the interval $[0, 10]$ and the initial conditions $y_1(0) = 3$ and $y_2(0) = 1$.

(ii) The equation for *simple harmonic motion* of a pendulum

$$my'' + ry' + D(y - l) = 0$$

on the interval $[0, 1]$ and $m = D = l = 1, r \in \{0, 1, 5\}$ as well as the initial conditions $y(0) = l$ and $y'(0) = 1$.

(iii) The equation for the *mathematical pendulum* (see Sheet 1 from Theory)

$$\phi'' = -({}^{g}\!/\!_{l}) \sin(\phi)$$

for $g = l = 1$, $y(0) = 0$ and $y'(0) \in \{1, 2, 4, 8\}$.

Write a short summary of your observations and the effects of the different parameters or initial values on the solution for each problem.

Project 2 (8 points)**.** Consider a grid of points $(x_i, y_i), i = 1, 2, \ldots, N$ on a rectangle $[a, b] \times [c, d] \subset \mathbb{R}^2$ and denote the step size in x- and y-direction by $dx, dy > 0$. This is realized in MATLAB by `[x,y] = meshgrid(a:dx:b,c:dy:d)` where `x` and `y` are matrices containing the x- and y-coordinates of the grid points. A discrete vectorfield defined by matrices `v` and `w`, where each grid point (x_i, y_i) is assigned to the vector (v_i, w_i) can be visualized by means of `quiver(x,y,v,w)`. An integral curve of the discrete vector field starting at a point (v_0, w_0) can be visualized by means of `streamline(x,y,v,w,v0,w0)`. The Matlab program `test_phase_diagram.m`, which you can download from the lecture homepage, implements this for a simple example. Modify the program to display the phase diagrams of the differential equations from Project 1 and at least two associated integral curves for each. Using your observations from Project 1, consider what are reasonable values for the intervals $[a, b]$ and $[c, d]$ under consideration.
Using your observations from Projects 1 and 2, explain how the phase diagrams of the differential equations are related to their solutions.