Abteilung für
Angewandte Mathematik

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Praktikum zur Vorlesung: Numerik für Differentialgleichungen – SoSe 2023

Sheet 2

Ausgabe: 17.05.2023, 12:00 Uhr | Abgabe: 24.05.2023, 12:00 Uhr

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Homepage to the lecture:

`https://aam.uni-freiburg.de/agsa/lehre/ss23/ndgln`

Project 1 (8 points)**.** The MATLAB-program `federpendel.m`, which you can download from the lecture homepage, realizes the explicit *Euler-Collatz method*, which is defined by the increment function $\Phi(t_k, y_k, \tau) = f(t_k + \frac{\tau}{2}, y_k + \tau f(t_k, y_k)/2)$, for the differential equation

$$y''(t) + ry'(t) + D(y(t) - l) = 0, \quad y(0) = y_0, \quad y'(0) = v_0. \tag{1}$$

(i) Investigate experimentally the dependence of the approximated solutions on the parameters r and D.

(ii) The exact solution for the special case $r = 1/10, D = 1, y_0 = l = 0, v_0 = 1$ is given by $y(t) = (v_0/\omega)e^{-rt/2}\sin(\omega t)$ where $\omega = (D - r^2/4)^{1/2}$. Compute the approximation error $|y_K - y(t_K)|$ for the step sizes $\tau = 2^{-s}, s \in \{1, 2, \ldots, 7\}$ at the time $t_K = 100$.

(iii) Modify the program to implement the explicit and implicit Euler method. Compare the qualitative behavior of the different approximated solutions for the time horizon T = 100. What do you notice for large step sizes?
Hint: When implementing the implicit Euler method, it is not sufficient to simply change the increment function in the program. Instead, solve the specific system of equations that results from applying the implicit Euler method to equation (1).

Project 2 (8 points)**.** The MATLAB-program `raeuber_beute.m`, which you can download from the lecture homepage, computes the approximate solution of the *Räuber-Beute-Modell* (see ex. 4 from sheet 1 for theory).

(i) Which numerical method is realized?

(ii) Test different step sizes and observe the qualitative behavior of the approximate solutions. For which step sizes do you get reasonable results?

(iii) Modify the program so that y_1 is approximated by the explicit and y_2 by the implicit Euler method. How does the qualitative behavior of the numerical solutions change?