Abteilung für
Angewandte Mathematik

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Praktikum zur Vorlesung: Numerik für Differentialgleichungen – SoSe 2023

Sheet 3

Ausgabe: 07.06.2023, 12:00 Uhr — Abgabe: 14.06.2023, 12:00 Uhr

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Homepage to the lecture:

`https://aam.uni-freiburg.de/agsa/lehre/ss23/ndgln`

Project 1 (8 points)**.** The MATLAB program `runge_kutta_expl.m`, which can be downloaded from the lecture homepage, implements the *explicit Runge-Kutta method* to solve the scalar initial value problem $y' = f(t, y), y(0) = y_0$.

(i) Document each line of the Phi subfunction. Which method is realized here?

(ii) The exact solution for $f(t, y) = -2y + 5\cos(t), y_0 = 2$ is given by $y(t) = 2\cos(t) + \sin(t)$. Determine the approximation error $|y(T) - y_K|$ at time $T = t_K = 10$, with step sizes $\tau = 2^{-s}, s = 0, 1, \ldots, 7$

(iii) Modify the program to implement the *explicit Euler method, the Euler-Collatz method, the classical Runge-Kutta method*, and the *3/8 rule*.

(iv) Determine for all methods the approximation errors $|y(T) - y_K|$ at time $T = 10$ with step sizes $\tau = 2^{-s}, s = 0, 1, \ldots, 7$. Plot them comparatively as polygonal lines in a graph with logarithmic y-axis scaling, which can be done in can be realized in Matlab with the command `semilogy`.

Project 2 (8 points)**.** (i) Modify the program from Project 1 to implement two MATLAB routines for the numerical approximation of ordinary differential equations with general *implicit Runge-Kutta methods*. Use a fixed point iteration on the one hand and the Newton method on the other hand. As termination criterion serves in each case the inequality $|\eta_{n+1} - \eta_n| < e_{max}$ for a reasonable bound $e_{max} > 0$.

(ii) Examine the respective iteration numbers in the time steps for the *Radau-3 method* using the example $y' = (1 + y^2)^{1/2}, y(0) = 0$ on the interval $[0, T] = [0, 4]$, whose exact solution is given by $y(t) = \sinh(t)$.