**Problems 1 and 2.** Let $a \in C(\mathbb{R}, \mathbb{R})$ be the softplus activation function and let $F\colon \mathbf{N} \to \cup_{d,m\in\mathbb{N}} C(\mathbb{R}^d, \mathbb{R}^m)$ be the function which satisfies for every $\Phi \in \mathbf{N}$ that

$$F(\Phi) = \mathcal{R}_a^{\mathbf{N}}(\Phi) \in C(\mathbb{R}^{\mathcal{I}(\Phi)}, \mathbb{R}^{\mathcal{O}(\Phi)}) \tag{1}$$

(cf. Definitions 1.2.8, 1.3.1, and 1.3.4). Prove or disprove that

1. $F$ is injective,
2. $F$ is surjective.

**Problem 3.** Let us define by

$$(\cdot) \star (\cdot)\colon \{(\Phi, \Psi) \in \mathbf{N} \times \mathbf{N}\colon \mathcal{I}(\Phi) = \mathcal{O}(\Psi)\} \to \mathbf{N} \tag{2}$$

the function which satisfies for all $\Phi, \Psi \in \mathbf{N}$, $k \in \{1, 2, \ldots, \mathcal{L}(\Phi) + \mathcal{L}(\Psi)\}$ with $\mathcal{I}(\Phi) = \mathcal{O}(\Psi)$ that $\mathcal{L}(\Phi \star \Psi) = \mathcal{L}(\Phi) + \mathcal{L}(\Psi)$ and

$$(\mathcal{W}_{k,\Phi\star\Psi}, \mathcal{B}_{k,\Phi\star\Psi}) = \begin{cases} (\mathcal{W}_{k,\Psi}, \mathcal{B}_{k,\Psi}) & : k \leq \mathcal{L}(\Psi) \\ (\mathcal{W}_{k-\mathcal{L}(\Psi),\Phi}, \mathcal{B}_{k-\mathcal{L}(\Psi),\Phi}) & : k > \mathcal{L}(\Psi) \end{cases} \tag{3}$$

(cf. Definitions 1.3.1 and 1.3.4). Prove or disprove that: For all $\Phi, \Psi \in \mathbf{N}$ with $\mathcal{I}(\Phi) = \mathcal{O}(\Psi)$ and $a \in C(\mathbb{R}, \mathbb{R})$ it holds that

$$\mathcal{R}_a^{\mathbf{N}}(\Phi \star \Psi) = [\mathcal{R}_a^{\mathbf{N}}(\Phi)] \circ [\mathcal{R}_a^{\mathbf{N}}(\Psi)]. \tag{4}$$

**Problem 4.** Exercise 2.2.2 in the lecture notes.