Problem 1. Employing composition of ANNs construct $\Phi \in \mathbf{N}$ such that

$$\mathcal{R}_{\mathfrak{r}}^{\mathbf{N}}(\Phi) : \mathbb{R}^3 \to \mathbb{R} \quad \text{and} \quad \mathcal{R}_{\mathfrak{r}}^{\mathbf{N}}(\Phi)(x_1, x_2, x_3) = \max\{x_1, x_2, x_3\}$$

where $\mathfrak{r}$ is the ReLU activation function. Calculate the number of parameters $\mathcal{P}(\Phi)$ of Φ.

Problem 2. Generalize Problem 1 to any $n \in \mathbb{N}$, i.e. construct $\Phi \in \mathbf{N}$ such that

$$\mathcal{R}_{\mathfrak{r}}^{\mathbf{N}}(\Phi)(x_1, \dots, x_n) = \max\{x_1, \dots, x_n\}$$

and calculate $\mathcal{P}(\Phi)$.

Problem 3. Prove that there exists $\Phi \in \mathbf{N}$ such that $\mathcal{R}_{\mathfrak{r}}^{\mathbf{N}}(\Phi) : \mathbb{R} \to \mathbb{R}$ and

$$\sup_{x \in [0,1]} \left| \mathcal{R}_{\mathfrak{r}}^{\mathbf{N}}(x) - x^2 \right| \leq \frac{1}{10}.$$

Problem 4. Generalize Problem 3 to any $\varepsilon > 0$, i.e. show that there exists $\Phi \in \mathbf{N}$ such that

$$\sup_{x \in [0,1]} \left| \mathcal{R}_{\mathfrak{r}}^{\mathbf{N}}(x) - x^2 \right| \leq \varepsilon.$$