Problem 1. Exercise 2.2.3

Problem 2. Construct a sum of ANNs of different length, i.e. let $\Phi_1, \ldots, \Phi_n$ satisfy $\mathcal{I}(\Phi_k) = \mathcal{I}(\Phi_1)$ and $\mathcal{O}(\Phi_k) = \mathcal{O}(\Phi_1)$ for all $k \in \{1, \ldots, n\}$. Let $a \in C(\mathbb{R}, \mathbb{R})$, $\mathbb{I}_1, \ldots, \mathbb{I}_n \in \mathbf{N}$ satisfy $\mathcal{I}(\mathbb{I}_j) = \mathcal{O}(\mathbb{I}_j) = \mathcal{O}(\Phi_j)$, $\mathcal{H}(\mathbb{I}_j) = 1$, and $\mathcal{R}_a^{\mathbf{N}}(\mathbb{I}_j)(x) = x$ for all $j \in \{1, \ldots, n\}$. Show that there exists $\Psi \in \mathbf{N}$ such that

$$\mathcal{R}_a^{\mathbf{N}}(\Psi) = \sum_{k=1}^{n} \mathcal{R}_a^{\mathbf{N}}(\Phi_k).$$

Problem 3. How does the dimension vector of Ψ in Problem 2 look like in terms of dimensions of $\Phi_1, \ldots, \Phi_n$?

Problem 4. Let $\Phi_1, \Phi_2 \in \mathbf{N}$ satisfy $\mathcal{I}(\Phi_1) = \mathcal{I}(\Phi_2) = m$, $\mathcal{O}(\Phi_1) = \mathcal{O}(\Phi_2) = n$, and $\mathcal{L}(\Phi_1) = \mathcal{L}(\Phi_2)$. Let $\mathcal{W}_1, \mathcal{W}_2 \in \mathbb{R}^{n \times n}$ and $\mathcal{B} \in \mathbb{R}^n$. Construct $\Psi \in \mathbf{N}$ such that for all $a \in C(\mathbb{R}, \mathbb{R})$ it holds that

$$\mathcal{R}_a^{\mathbf{N}}(\Psi) = \mathcal{W}_1 \mathcal{R}_a^{\mathbf{N}}(\Phi_1) + \mathcal{W}_2 \mathcal{R}_a^{\mathbf{N}}(\Phi_2) + \mathcal{B}.$$