Abteilung für
Angewandte Mathematik

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Introduction to Theory and Numerics of PDEs – WiSe 2023/2024

Sheet 10

Ausgabe: 22.01.2024, 12:00 Uhr Abgabe: 29.01.2024, 12:00 Uhr

Homepage to the lecture:

`https://aam.uni-freiburg.de/agsa/lehre/ws23/tun0`

Exercise 1 (4 points)**.** We consider mesh $\mathcal{M}$ of the interval $\Omega = [a, b]$ with nodes $\{a = x_0 < x_1 < \dots x_M = b\}$. As in the lecture fix the set of piecewise linear functions

$$S^0_{1,0}(\mathcal{M}) = \{v \in C([a, b]) \,|\, v|_{[}x_{i-1}, x_{i]} \text{ is linear } , i = 1, \dots, M, v(a) = v(b) = 0\}$$

and the set of tent functions $\{\phi^1, \phi^2, \dots, \phi^{M-1}\}$ given by

$$\phi^j(x) = \begin{cases} \frac{x - x_{j-1}}{x_j - x_{j-1}}, & x_{j-1} \le x \le x_j \\ \frac{x_{j+1} - x}{x_{j+1} - x_j}, & x_j \le x \le x_{j+1}. \end{cases}$$

Show that $\{\phi^1, \phi^2, \dots, \phi^{M-1}\}$ is a basis for $S^0_{1,0}(\mathcal{M})$.

Exercise 2 (4 points)**.** Prove that there is exactly one plane through three non-collinear points in $\mathbb{R}^3$.

Exercise 3 (4 points)**.** Let $u \in C^3(\Omega)$. Show that we have

$$|\Delta u|^2 - |D^2 u|^2 = \operatorname{div}(\nabla u \Delta u - \frac{1}{2}\nabla|\nabla u|^2).$$

Exercise 4 (4 points)**.** The mesh displayed in the picture contains a few *hanging nodes* marked with red dots. Sketch a triangulation (without hanging nodes) with the same nodes as that mesh M.

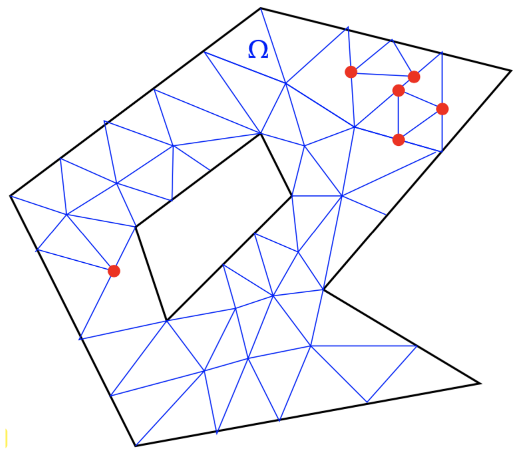

FIG. 1. Mesh M