Abteilung für
Angewandte Mathematik

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## Introduction to Theory and Numerics of PDEs – WiSe 2023/2024

Sheet 8

Ausgabe: 08.01.2024, 12:00 Uhr    Abgabe: 15.01.2024, 12:00 Uhr

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**Homepage to the lecture:**

`https://aam.uni-freiburg.de/agsa/lehre/ws23/tun0`

**Exercise 1** (4 points)**.** Let $(v_j)_{j\in\mathbb{N}} \subset \ell^2(\mathbb{N})$ be defined by $v_{j,n} = \delta_{j,n}$ i.e.,

$$v_j = (0, \dots, 0, 1, 0, \dots).$$

Prove that the sequence converges weakly and determine the weak limit.

**Exercise 2** (2 + 2 points)**.** (i) Prove that for $1 < p, q > \infty$ with $1/p + 1/q = 1$ and all $a, b \in \mathbb{R}_{\geq 0}$ we have

$$ab \leq \frac{1}{p}a^p + \frac{1}{q}b^q.$$

(ii) Prove Hölder's inequality

$$\int_\Omega |uv| \text{ dx} \leq ||u||_{L^p(\Omega)}||v||_{L^q(\Omega)}$$

**Exercise 3** (2 + 2)**.** Let $d \in \mathbb{N}, s \in \mathbb{R}$ and $\Omega = B_1(0) \subset \mathbb{R}^d$, and define $u(x) = |x|^s$ for $x \in \Omega \setminus \{0\}$.

(i) Determine all $s \in \mathbb{R}$ so that $u \in L^p(\Omega)$.
(ii) Determine all $s \in \mathbb{R}$ so taht $u \in W^{1,p}(\Omega)$.

**Exercise 4** (4 points)**.** Prove that $|a + b|^p \leq |a + b|^{p-1}(|a| + |b|)$ for all $a, b \in \mathbb{R}$ and use Hölder's inequality to deduce Minkowski's inequality

$$||u + v||_{L^p(\Omega)} \leq ||u||_{L^p(\Omega)}\,||v||_{L^p(\Omega)}$$