Abteilung für
Angewandte Mathematik

JProf. Dr. Diyora Salimova
M.Sc. Mario Keller

Introduction to Theory and Numerics of PDEs – WiSe 2023/2024

Sheet 9

Ausgabe: 15.01.2024, 12:00 Uhr — Abgabe: 22.01.2024, 12:00 Uhr

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Homepage to the lecture:

`https://aam.uni-freiburg.de/agsa/lehre/ws23/tun0`

Exercise 1 (4 points)**.** Let $u, v \in W^{1,2}(\Omega)$. Prove that $uv \in W^{1,2}(\Omega)$ with $\nabla(uv) = u\nabla v + v\nabla u$.

Exercise 2 (2 + 2 points)**.**

(i) Derive a weak formulation for the boundary value problem

$$\begin{aligned} -\mathrm{div}(K\nabla u) + b \cdot \nabla u + cu &= f \qquad \text{in } \Omega, \\ u &= u_D \qquad \text{on } \Gamma_D, \\ (K\nabla u) &= g \qquad \text{on } \Gamma_N. \end{aligned}$$

(ii) Specify conditions on the coefficients that lead to the existence of a unique weak solution $u \in H^1(\Omega)$.

Exercise 3 (4 points)**.** Let $\Omega = (0,1)^2$ and define for $j, k = 1, 2, \ldots, N$,

$$\phi_{j,k}(x_1, x_2) = \sin(\pi x_1 j/N)\sin(\pi x_2 k/N)$$

and let V_h be the span of $(\phi_{j,k} \,|\, j, k = 1, 2, \ldots, N)$. Compute the stiffness matrix for the bilinear mapping related to the Laplace operator.

Exercise 4 (4 points)**.** Let $Q' \subset \mathbb{R}^{d-1}$ be open and let $h \in C^2(Q')$ be concave. Prove that for all $x' \in Q'$ we have

$$\sum_{i=1}^{d-1} \partial_i^2 h(x') \le 0.$$