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Sheet 2

1. Let $F\colon \mathbb{R} \to [0,1]$ be a distribution function and let $I_F\colon (0,1) \to \mathbb{R}$ be the generalized inverse distribution function associated to F. Prove that for all open sets $D \subseteq \mathbb{R}$ with the property that $F|_D\colon D \to [0,1]$ is injective and all $y \in F(D)$: $F|_D^{-1}(y) = I_F(y)$.

2. Let $a, b \in \mathbb{R}$ be real numbers with $a < b$ and let $F\colon \mathbb{R} \to [0,1]$ be a distribution function which satisfies for all $y \in (0,1)$ that

$$I_F(y) = yb + (1-y)a. \tag{1}$$

Specify $F(x)$, $x \in \mathbb{R}$, explicitly and prove that your result is correct.

3. Prove or disprove the following statement: For all infinitely often differentiable functions $f\colon [0,1] \to \mathbb{R}$ it holds that

$$\inf_{n\in\mathbb{N}} \left(n^2 \left| T^n_{[0,1]}[f] - \textstyle\int_0^1 f(x)\,dx \right| \right) = 0, \tag{2}$$

where $T^n_{[0,1]}$ is the trapezoidal rule.

Due: Monday, 04.11.2024.
Webpage: `https://aam.uni-freiburg.de/agsa/lehre/ws24/numsde/index.html`