

Problems 1 and 2. Let $a \in C(\mathbb{R}, \mathbb{R})$ be the softplus activation function and let $F: \mathbf{N} \rightarrow \cup_{d,m \in \mathbf{N}} C(\mathbb{R}^d, \mathbb{R}^m)$ be the function which satisfies for every $\Phi \in \mathbf{N}$ that

$$F(\Phi) = \mathcal{R}_a^{\mathbf{N}}(\Phi) \in C(\mathbb{R}^{\mathcal{I}(\Phi)}, \mathbb{R}^{\mathcal{O}(\Phi)}) \quad (1)$$

(cf. Definitions 1.2.8, 1.3.1, and 1.3.4). Prove or disprove that

1. F is injective,
2. F is surjective.

Problem 3. Let us define by

$$(\cdot) \star (\cdot): \{(\Phi, \Psi) \in \mathbf{N} \times \mathbf{N}: \mathcal{I}(\Phi) = \mathcal{O}(\Psi)\} \rightarrow \mathbf{N} \quad (2)$$

the function which satisfies for all $\Phi, \Psi \in \mathbf{N}$, $k \in \{1, 2, \dots, \mathcal{L}(\Phi) + \mathcal{L}(\Psi)\}$ with $\mathcal{I}(\Phi) = \mathcal{O}(\Psi)$ that $\mathcal{L}(\Phi \star \Psi) = \mathcal{L}(\Phi) + \mathcal{L}(\Psi)$ and

$$(\mathcal{W}_{k, \Phi \star \Psi}, \mathcal{B}_{k, \Phi \star \Psi}) = \begin{cases} (\mathcal{W}_{k, \Psi}, \mathcal{B}_{k, \Psi}) & : k \leq \mathcal{L}(\Psi) \\ (\mathcal{W}_{k - \mathcal{L}(\Psi), \Phi}, \mathcal{B}_{k - \mathcal{L}(\Psi), \Phi}) & : k > \mathcal{L}(\Psi) \end{cases} \quad (3)$$

(cf. Definitions 1.3.1 and 1.3.4). Prove or disprove that: For all $\Phi, \Psi \in \mathbf{N}$ with $\mathcal{I}(\Phi) = \mathcal{O}(\Psi)$ and $a \in C(\mathbb{R}, \mathbb{R})$ it holds that

$$\mathcal{R}_a^{\mathbf{N}}(\Phi \star \Psi) = [\mathcal{R}_a^{\mathbf{N}}(\Phi)] \circ [\mathcal{R}_a^{\mathbf{N}}(\Psi)]. \quad (4)$$

Additionally, **Exercises 1.3.1, 1.3.2, 1.3.3** from the lecture notes.