

Mathematical Introduction to Deep Neural Networks

Exercise Sheet 5

Exercise 1. Show that there exists an ANN $\Phi \in \mathbf{N}$ such that $\mathcal{R}_{\mathfrak{r}}^{\mathbf{N}}: \mathbb{R}^3 \rightarrow \mathbb{R}$ and for all $x = (x_1, x_2, x_3) \in \mathbb{R}^3$ it holds that

$$(\mathcal{R}_{\mathfrak{r}}^{\mathbf{N}}(\Phi))(x) = \max\{x_1, x_2, x_3\}. \quad (1)$$

Here $\mathfrak{r}$ is a ReLU activation function. Estimate the number of parameters of Φ .

Exercise 2. Show that there exists an ANN $\Phi \in \mathbf{N}$ such that $\mathcal{R}_{\mathfrak{r}}^{\mathbf{N}}: \mathbb{R}^3 \rightarrow \mathbb{R}$ and for all $x = (x_1, x_2, x_3) \in \mathbb{R}^3$ it holds that

$$(\mathcal{R}_{\mathfrak{r}}^{\mathbf{N}}(\Phi))(x) = \min\{x_1, x_2, x_3\}. \quad (2)$$

Here $\mathfrak{r}$ is a ReLU activation function. Estimate the number of parameters of Φ .

Exercise 3. Show that there exists an ANN $\Phi \in \mathbf{N}$ such that $\mathcal{R}_{\mathfrak{r}}^{\mathbf{N}}: \mathbb{R}^3 \rightarrow \mathbb{R}^2$ and for all $x = (x_1, x_2, x_3) \in \mathbb{R}^3$ it holds that

$$(\mathcal{R}_{\mathfrak{r}}^{\mathbf{N}}(\Phi))(x) = (\max\{x_1, x_2, x_3\}, \min\{x_1, x_2, x_3\}). \quad (3)$$

Here $\mathfrak{r}$ is a ReLU activation function. Estimate the number of parameters of Φ .

Additionally, **Exercises 2.2.3** in the lecture notes.