Correction to the publication

Error analysis of a finite element method for the Willmore flow of graphs

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In Theorem 2.2 the choice of the discrete initial value u_{0h} needs to be modified in order to guarantee higher order convergence for $e_w(0) = \widehat{w}_h(0) - w_h(0)$. Instead of choosing the initial value u_{0h} as the minimal surface projection $\widehat{u}_{0h}$ of the continuous initial value u_0 according to (2.17), we proceed as follows: Let $\widehat{w}_{0h}$ be the solution of (2.22) at time $t = 0$. We then define the discrete initial value u_{0h} as the solution of the equation

$$\int_\Omega \frac{\nabla u_{0h} \cdot \nabla \varphi_h}{Q_{0h}} = \int_\Omega \frac{\widehat{w}_{0h}}{\widehat{Q}_{0h}} \varphi_h \qquad (0.1)$$

for all $\varphi_h \in X_{h0}$. Here $Q_{0h} = \sqrt{1 + |\nabla u_{0h}|^2}$ and similarly $\widehat{Q}_{0h} = \sqrt{1 + |\nabla \widehat{u}_{0h}|^2}$.
With the methods and results from the paper it is easy to prove the following estimates. These need to be inserted on page 37 at two places where $e_u(0)$ and $e_w(0)$ are used. The bounds in Theorem 2.2 are not affected by this modification.

Lemma 0.1. *For $e_u(0) = \widehat{u}_{0h} - u_{0h}$ and $e_w(0) = \widehat{w}_{0h} - w_{0h}$ we have the estimates*

$$\|e_u(0)\|_{H^1} + \|e_w(0)\| \le ch^2 |\log h|.$$

Proof. The minimal surface projection was defined in (2.17). Together with (0.1) this leads to the error relation

$$\int_\Omega \left(\frac{\nabla \widehat{u}_{0h}}{\widehat{Q}_{0h}} - \frac{\nabla u_{0h}}{Q_{0h}} \right) \cdot \nabla \varphi_h = \int_\Omega \left(\frac{w_0}{Q_0} - \frac{\widehat{w}_{0h}}{\widehat{Q}_{0h}} \right) \varphi_h, \qquad \varphi_h \in X_{h0}.$$

Here w_0 and Q_0 are w and Q at time $t = 0$. The choice $\varphi_h = \widehat{u}_{0h} - u_{0h} = e_u(0)$ leads to

$$\int_\Omega \left(\frac{\nabla \widehat{u}_{0h}}{\widehat{Q}_{0h}} - \frac{\nabla u_{0h}}{Q_{0h}} \right) \cdot \nabla e_u(0) = \int_\Omega \left(\frac{1}{Q_0} - \frac{1}{\widehat{Q}_{0h}} \right) w_0 e_u(0) + \int_\Omega \frac{1}{\widehat{Q}_{0h}} (w_0 - \widehat{w}_{0h}) e_u(0).$$

Using a simplified version of the analysis in Lemma 3.2 together with Lemma A.1 it is not difficult to obtain the estimate

$$\|\nabla e_u(0)\|^2 \le ch^4 |\log h|^2. \qquad (0.2)$$

Next, combining (0.1) with equation (2.11) at time $t = 0$ we can conclude that

$$\int_\Omega \frac{w_{0h}}{Q_{0h}} \varphi_h = \int_\Omega \frac{\widehat{w}_{0h}}{\widehat{Q}_{0h}} \varphi_h$$

for every $\varphi_h \in X_{h0}$. The choice $\varphi_h = \widehat{w}_{0h} - w_{0h}$ together with (0.2) then implies that $\|e_w(0)\| \le ch^2 |\log h|$ and the lemma is proved. ∎

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