L^p-versions of the generalized Korn inequalities for incompatible tensor fields

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December 13, 2019

For $1 < p < \infty$ we prove an L^p-version of the generalized Korn's inequality for incompatible tensor fields P in $W_0^{1,p}(\mathrm{Curl};\Omega,\mathbb{R}^{3\times 3})$. More precisely, let $\Omega \subset \mathbb{R}^3$ be a bounded Lipschitz domain. Then there exists a constant $c > 0$ such that

$$\|P\|_{L^p(\Omega,\mathbb{R}^{3\times 3})} \leq c \left(\|\mathrm{sym}\, P\|_{L^p(\Omega,\mathbb{R}^{3\times 3})} + \|\mathrm{Curl}\, P\|_{L^p(\Omega,\mathbb{R}^{3\times 3})}\right)$$

holds for all tensor fields $P \in W_0^{1,p}(\mathrm{Curl};\Omega,\mathbb{R}^{3\times 3})$, i.e., for all $P \in W^{1,p}(\mathrm{Curl};\Omega,\mathbb{R}^{3\times 3})$ with vanishing tangential trace $P \times \nu = 0$ on $\partial\Omega$ where ν denotes the outward unit normal vector field to $\partial\Omega$.
For compatible $P = \boldsymbol{\nabla} u$ this recovers an L^p-version of the classical Korn's first inequality

$$\|\boldsymbol{\nabla} u\|_{L^p(\Omega,\mathbb{R}^{3\times 3})} \leq c\, \|\mathrm{sym}\, \boldsymbol{\nabla} u\|_{L^p(\Omega,\mathbb{R}^{3\times 3})} \quad \text{with } \boldsymbol{\nabla} u \times \nu = 0 \quad \text{on } \partial\Omega,$$

and for skew-symmetric $P = A \in \mathfrak{so}(3)$ an L^p-version of the Poincaré inequality

$$\|A\|_{L^p(\Omega,\mathfrak{so}(3))} \leq c\, \|\mathrm{Curl}\, A\|_{L^p(\Omega,\mathbb{R}^{3\times 3})} \quad \text{with } A \times \nu = 0 \;\Leftrightarrow\; A = 0 \quad \text{on } \partial\Omega.$$

Further generalizations will be discussed.

AMS 2010 subject classification: Primary: 35A23; Secondary: 35B45, 35Q74, 46E35.

Keywords: $W^{1,p}(\mathrm{Curl})$-Korn's inequality, Poincaré's inequality,Curl-spaces, gradient plasticity, dislocation density, relaxed micromorphic model

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