9th GAMM-Workshop on Analysis of Partial Differential Equations

Book of Abstracts

Institute: Albert-Ludwigs-Universität Freiburg

Department of Applied Mathematics

Date: October 06-08, 2021

"Ein Mathematiker ist eine Maschine, die Kaffee in Theoreme umwandelt" Alfréd Rényi

Contents

Chapter 1 Plenary talks

Elisabetta Chiodaroli, University of Pisa

Wild solutions to isentropic Euler equations starting from smooth initial data

In this talk we consider the isentropic Euler equations of gas dynamics in the whole two-dimensional space and we prove the existence of a smooth initial datum which admits infinitely many bounded admissible weak solutions.

Cristiana De Filippis, University of Torino

Perturbations beyond Schauder

Schauder estimates hold in the nonuniformly elliptic setting. Specifically, first derivatives of solutions to nonuniformly elliptic variational problems and elliptic equations are locally Hölder continuous, provided coefficients are locally Hölder continuous. In this talk, based on papers [1, 2, 3], I will present new regularity results for minima of nonuniformly elliptic functionals with emphasis on new borderline regularity criteria.

References:
[1] C. De Filippis, Quasiconvexity and partial regularity via nonlinear potentials. Preprint (2021).
[2] C. De Filippis, G. Mingione, Lipschitz bounds and nonautonomous integrals. Arch. Ration. Mech. Anal., to appear.
[3] C. De Filippis, G. Mingione, Nonuniformly elliptic Schauder estimates. Preprint (2021).

Carolin Kreisbeck, Katholische Universität Eichstatt-Ingolstadt

Nonlocal variational problems: Structure-preservation during relaxation?

Nonlocal variational problems arise in various applications, such as continuum mechanics, the theory of phase transitions, or image processing. Naturally, the presence of nonlocalities leads to new effects, and the standard methods in the calculus of variations, which tend to rely intrinsically on localization arguments, do not apply. This this talk addresses relaxation of two classes of functionals - double-integrals and nonlocal supremals. Our focus lies on the question of whether the resulting relaxed functionals preserve their structure. We give an affirmative answer for nonlocal supremals in the scalar setting, along with a closed representation formula in terms of separate level convexification of a suitably diagonalized supremand, and discuss results in the vectorial case. Regarding double-integrals, a full understanding of the problem is still missing. We present the first counterexample showing that weak lower semicontinuous envelopes fail to be double-integrals in general. On a technical level, both findings rely on a characterization of the asymptotic behavior of (approximate) nonlocal inclusions, a theoretical result of independent interest.

This is based on joint work with Elvira Zappale (Sapienza University of Rome) and Antonella Ritorto (KU Eichstätt-Ingolstadt).

Stefan Neukamm, Technische Universität Dresden

Quantitative homogenization in non-linear elasticity: periodic composites & random laminates

We discuss some classical and recent results in quantitative homogenization of nonlinearly elastic materials with periodic or random microstructure. The central object that we discuss is the homogenized stored energy function, which is a priori rather implicitly defined. We show that for periodic composites or random laminates a corrector-theory is available. In particular, for the case of random laminates, we present error estimates on the approximation of the stored energy function, the stress tensor and tangent moduli via periodic representative volume elements.

The talk is based on joint work with M. Schäffner and M. Varga.

Andrea Pinamonti, University of Trento

Γ-convergence and H-convergence for integral functionals and differential operators depending on vector fields

Given a family of locally Lipschitz vector fields $X(x) = (X_1(x), \ldots, X_m(x))$ on $\mathbb{R}^n$, $m \leq n$, we study integral functionals depending on X. We will discuss some results of Γ-convergence and we will apply them to study the H-convergence of linear differential operators in divergence form modeled on X.
The talk is based on joint works with Alberto Maione, Francesco Serra Cassano, Fares Essebei and Simone Verzellesi.

Christophe Prange, Cergy Paris Université

Quantitative regularity for the Navier-Stokes equations via spatial concentration

This talk is concerned with the regularity of solutions to the three-dimensional Navier-Stokes equations under boundedness of certain critical or slightly supercritical norms. I will explain recent ideas to quantify explicitly some regularity criteria. One of the keys is to study the concentration of specific critical quantities. This is joint work with Tobias Barker (University of Bath).

Patrizia Pucci, University of Perugia

Fractional elliptic systems with critical nonlinearities

Elliptic systems arise in biological applications (e.g. population dynamics) or physical applications (e.g. models of a nuclear reactor) and have been drawn a lot of attention. In the nonlocal case, there are not so many papers on weakly coupled systems in $\mathbb{R}^N$. In this talk we present some recent existence, uniqueness and multiplicity results for positive solutions of a class of weakly coupled nonlocal systems of equations in $\mathbb{R}^N$, which are new also in the local case. Moreover, we also provide a global compactness result, which gives a complete description of the Palais-Smale sequences of the treated systems. To the best of our knowledge, this decomposition has been studied only for systems of equations in bounded domains.

The results of the talk are based on joint works with M. Bhakta, S. Chakraborty and O.H. Miyagaki.

Angkana Rüland, University of Heidelberg

On Instability Mechanisms in Inverse Problems

Many PDE driven inverse problems are notoriously ill-posed. In this talk I will discuss three robust instability mechanisms leading to instability. These are based on strong global, weak global and only microlocal smoothing properties of the associated forward operators. As a consequence, these mechanism are applicable to a wide range of inverse problems including elliptic, parabolic and hyperbolic ones.

The talk is based on joint work with Herbert Koch (University of Bonn) and Mikko Salo (University of Jyväskylä).

Julian Scheuer, Cardiff University

Stability from rigidity via umbilicity

The soap bubble theorem says that a closed, embedded surface of the Euclidean space with constant mean curvature must be a round sphere. Especially in real-life problems it is of importance whether and to what extent this phenomenon is stable, i.e. when a surface with almost constant mean curvature is close to a sphere. This problem has been receiving lots of attention until today, with satisfactory recent solutions due to Magnanini/Pogessi and Ciraolo/Vezzoni. The purpose of this talk is to discuss further problems of this type and to provide two approaches to their solutions. The first one is a new general approach based on stability of the so-called "Nabelpunktsatz". The second one is of variational nature and employs the theory of curvature flows.

Chapter 2 Contributed talks

Anna Balci, Bielefeld University

New on analysis and numerics for non-autonomous variational problems with Lavrentiev gap

New examples on Lavrentiev phenomenon using fractal contact sets are construscted. Comparing to the well-known examples of Zhikov it is not important that at the saddle point the variable exponent crosses the threshold dimension. As a consequence we give the negative answer to the well-known conjecture that the dimension plays a critical role for the Lavrentiev gap to appear. We apply our method to the setting of variable exponents, the double phase potential and weighted p-energy. We investigate a Crouzeix- Raviart FEM for variational problems with non-autonomous integrands and non-standard growth. While conforming schemes fail due to the Lavrentiev gap phenomenon, we prove that the solution to this non-conforming scheme converges to the global minimiser. The talk is based on several projects with Lars Diening, Mikhail Surnachev, Christoph Ortner and Johannes Storn.

Acknowledgments: This research was Funded by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) SFB 1283/2 2021 317210226.

Luca Benatti, University of Trento

Minkowski Inequality in Riemannian manifolds with nonnegative Ricci curvature

In this talk we present the sharp Minkowski Inequality in the setting of complete Riemannian manifold (M, g) of dimension $n \geq 3$, with nonnegative Ricci curvature and Euclidean Volume Growth, proved in a recent paper with M. Fogagnolo and L. Mazzieri. In particular, we show that in this framework the following inequality

$$\left(\frac{|\partial\Omega^*|}{|\mathbb{S}^{n-1}|}\right)^{\frac{n-2}{n-1}} \mathrm{AVR}(g)^{\frac{1}{n-1}} \leq \frac{1}{|\mathbb{S}^{n-1}|} \int\limits_{\partial\Omega} \left|\frac{\mathrm{H}}{n-1}\right| \mathrm{d}\sigma, \tag{2.1}$$

holds for every bounded open subset $\Omega \subseteq M$ with smooth boundary, where H is the mean curvature of the boundary, Ω^* the strictly outward minimising hull of Ω and $\mathrm{AVR}(g)$ is the Asymptotic Volume Ratio of (M, g). The proof is based on new Monotonicity Formulas, that, despite the mild regularity of p-harmonic functions, are in force at each level set of the p-capacitary potential associated to Ω. Exploiting this knowledge and using a contradiction argument built on the Iso-p-capacitary inequality

$$\frac{\mathrm{Cap}_p(\mathbb{B}^n)^n}{|\mathbb{B}^n|^{n-p}} \mathrm{AVR}(g)^p \leq \frac{\mathrm{Cap}_p(\Omega)^n}{|\Omega|^{n-p}},$$

we derive an L^p-version of (1). The Minkowski Inequality is then obtained sending $p \to 1^+$.

Maicol Caponi, Technische Universität Dresden

An existence result for the fractional Kelvin-Voigt's model on time-dependent cracked domains

In this talk I present an existence result for the fractional Kelvin-Voigt's model involving Caputo's derivative on time-dependent cracked domains. More precisely, I consider the following dynamic system

$$\ddot{u}(t) - \operatorname{div}(\mathbb{C}eu(t) + \mathbb{B}D_t^\alpha eu(t)) = f(t) \quad \text{in } \Omega \setminus \Gamma_t, \quad t \in (0,T).$$

In the equation above, $\Omega \subset \mathbb{R}^d$ represents the reference configuration of the material, the set $\Gamma_t \subset \Omega$ models the crack at time t (which is prescribed), $u(t)\colon \Omega \setminus \Gamma_t \to \mathbb{R}^d$ is the displacement of the deformation, and $f(t)$ is the forcing term. Moreover, $\mathbb{C}$ and $\mathbb{B}$ are two positive tensors acting on the space of symmetric matrices, $eu(t)$ denotes the symmetric part of the gradient of the function $u(t)$, and D_t^α is the Caputo derivative of order $\alpha \in (0,1)$. I first show the existence of a solution to a regularized version of this model by using a time-discretization procedure. Then, I derive some uniform estimates and, thanks a compactness argument, I prove that the fractional Kelvin-Voigt's model admits a solution that satisfies an energy-dissipation inequality. Finally, I show that the solution is unique when the crack is not moving.

This is a joint work with F. Sapio.

Giovanni Covi, University of Heidelberg

The higher order fractional Calderon problem with local perturbations

I will present an inverse problem for the fractional Schrödinger equation with a local perturbation by a linear partial differential operator. I will show unique recoverability of the coefficients of the perturbation from the Dirichlet-to-Neumann map associated to the perturbed equation. This is proved for two classes of coefficients: coefficients which belong to certain spaces of Sobolev multipliers and coefficients which belong to fractional Sobolev spaces with bounded derivatives. This study generalizes recent results for the zeroth and first order perturbations to higher order. This talk is based on recent work of mine with K. Mönkkönen, J. Rallo and G. Uhlmann: https://arxiv.org/abs/2008.10227

Fares Essebei, University of Trento

Variational problems concerning sub-Finsler metrics in Carnot Groups

The paper is devoted to the study of geodesic distances defined on a subdomain of a given Carnot group, which are bounded both from above and from below by fixed multiples of the CarnotCarathédory distance. We show that the uniform convergence (on compact sets) of these distances can be equivalently characterized in terms of Γ-convergence of several kinds of variational problems. Moreover, we investigate the relation between this class of distances and the sub-Finsler convex metrics defined on the horizontal bundle.

Mattia Fogagnolo, Scuola Normale Superiore of Pisa

The strictly outward minimizing hull in Riemannian manifolds and its relation with p-Capacities

This talk deals with some of the results contained in a recent work with L. Mazzieri. We analyze the notion of strictly outward minimizing hull of bounded sets Ω in complete Riemannian manifolds, that can be characterized as the maximal volume solution to the least area problem with obstacle Ω. We find sufficient geometric conditions on the ambient manifold ensuring the well posedness of such optimal envelope, that in general cannot be expected to exist. Moreover, under such conditions, we also show that the perimeter of the strictly outward minimizing hull can be recovered as limit of p-capacities as $p \to 1^+$. It is well-known that the p-capacity is realized as L^p norm of the gradient of the p-harmonic potential of Ω, and this sets in particular a link with nonlinear potential theory. In this talk we will focus on manifolds admitting a global positive isoperimetric constant, a condition flexible enough to include the important classes of manifolds with nonnegative Ricci curvature and Euclidean volume growth and Cartan-Hadamard manifolds.

Paul Hüttl, University of Regensburg

Phasefield methods for spectral shape and topology optimization

We optimize a selection of eigenvalues of the Laplace operator with Dirichlet or Neumann boundary conditions by adjusting the shape of the domain on which the eigenvalue problem is considered. Here, a phasefield function is used to represent the shapes over which we minimize. The idea behind this method is to modify the Laplace operator by introducing phasefield dependent coefficients in order to extend the eigenvalue problem on a fixed design domain containing all admissible shapes. The resulting shape and topology optimization problem can then be formulated as an optimal control problem with PDE constraints in which the phasefield function acts as the control. For this optimal control problem, we establish firstorder necessary optimality conditions and we rigorously derive its sharp interface limit. Eventually, we present and discuss several numerical simulations for our optimization problem.

Patrik Knopf, University of Regensburg

On multiphase Cahn-Hilliard-Brinkman/Darcy models for stratified tumor growth with chemotaxis

We consider a multiphase Cahn-Hilliard model for tumor growth with general source terms. The multiphase approach allows us to consider multiple cell types and multiple chemical species (oxygen and/or nutrients) that are consumed by the tumor. Compared to classical two-phase tumor growth models, the multiphase model can be used to describe a stratified tumor exhibiting several layers of tissue (e.g., proliferating, quiescent and necrotic tissue) more precisely. Our model consists of a convective Cahn-Hilliard type equation to describe the tumor evolution, a velocity equation for the associated volume-averaged velocity field, and a convective reaction- diffusion type equation to describe the density of the chemical species. The velocity equation is either represented by Darcy's law or by the Brinkman equation. We further discuss the existence of global weak solutions to these models as well as the "Darcy limit" where the positive viscosities in the Brinkman equation are sent to zero.

Tobias König, Institut de Mathématiques de Jussieu - Paris Rive Gauche

Constant Q-curvature metrics with a singularity

For dimensions $n \geq 3$, we classify singular solutions to the generalized Liouville equation $(-\Delta)^{\frac{n}{2}} u = e^{nu}$ on $\mathbb{R}^n \setminus \{0\}$ with the finite integral condition $\int_{\mathbb{R}^n} e^{nu} < \infty$ in terms of their behavior at 0 and ∞. These solutions correspond to metrics of constant Q-curvature which are singular in the origin. Conversely, we give an optimal existence result for radial solutions. This extends some recent results on solutions with singularities of logarithmic type to allow for singularities of arbitrary order. As a key tool to the existence result, we derive a new weighted Moser-Trudinger inequality for radial functions.

This is joint work with Paul Laurain (IMJ-PRG Paris and ENS Paris).

Jona Lelmi, University of Bonn

Large data limit of the MBO scheme for data clustering

The MBO scheme is an efficient algorithm for approximating mean curvature flow. Recently, Bertozzi et al. adapted it to the setting of graphs, where it is used for data clustering. Given some data, one constructs the similarity graph associated to the data points. The goal is to split the data in two meaningful clusters. The algorithm produces the clusters by alternating between diffusion on the graph and pointwise thresholding. After a suitable stopping criterion, one is left with one cluster (and its complement). It turns out that this can be thought of as a local minimizer of the thresholding energy on the graph - which is defined by averaging the diffusion of the first step over points belonging to the complement of the diffused cluster. It is then natural to ask the following question: assume that the data points are sampled from a weighted Riemannian submanifold of some Euclidean space, does the thresholding energy on the similarity graph converge to a suitable continuum limit as the number of data points goes to infinity and the step size converges to zero? We will give a partial answer to the question. In particular, we identify the limit obtained by first letting the number of data points go to infinity and then taking the step size to zero. This is ongoing joint work with Tim Laux.

Greta Marino, Technische Universität Chemnitz

Existence of weak solutions to a crossdiffusion CahnHilliard system

In this talk we study a CahnHilliard model for a multicomponent mixture with cross diffusion effects, degenerate mobility and where only one of the species does separate from the others. We define a notion of weak solution adapted to possible degeneracies and our main result is (global in time) existence. In order to overcome the lack of apriori estimates, our proof uses the formal gradient flow structure of the system and an extension of the boundedness by entropy method which involves a careful analysis of an auxiliary variational problem. This allows to obtain solutions to an approximate, timediscrete system. Letting the time step size go to zero, we recover the desired weak solution where

Alice Marveggio, Institute of Science and Technology Austria

Rates of convergence of the vector-valued Allen-Cahn equation towards multiphase mean curvature flow

The vector-valued Allen-Cahn equation with a multi-well potential is expected to approximate multiphase mean curvature flow in the limit of vanishing interface thickness. However, to the best of our knowledge, proving an unconditional convergence result for such a diffuse interface approximation for multiphase mean curvature flow has remained an open problem. We present a proof for rates of convergence of the vector-valued Allen-Cahn equation towards multiphase mean curvature flow in the planar case d=2, assuming that a classical (smooth) solution to the latter exists. The result is valid for suitable multi-well potentials and starting from well-prepared initial data. Our approach is based on a relative entropy technique inspired by [2] and, in particular, on the notion of gradient-flow calibration introduced in [1]. Beyond the planar setting, our results also apply to three-dimensional double bubbles (or more generally to evolving interfaces without quadruple junctions), relying on the existence of the gradient-flow calibration shown in [3].

[1] J. Fischer, S. Hensel, T. Laux, T. Simon. The local structure of the energy landscape in multiphase mean curvature flow: Weak-strong uniqueness and stability of evolutions. Preprint, 2020. [2] J. Fischer, T. Laux, T. Simon. Convergence rates of the Allen-Cahn equation to mean curvature flow: A short proof based on relative entropies. SIAM J. Math. Anal., 52(6):6222 6233, 2020. [3] S. Hensel and T. Laux. Weak-strong uniqueness for the mean curvature flow of double bubbles. Preprint, 2021.

David Padilla-Garza, Technische Universität Dresden

Connecting energy to irregularity in prestrained elastic sheets

Prestrained elastic sheets are thin structures in which the ground state is not a rigid motion, but is instead characterized by a preferred metric varying with position. Such prestrain can be caused by adding inclusions to an already existing sheet, or by heating a sheet that is built to swell differentially throughout; it is also thought to occur by natural growth. Energy minimization drives such a sheet to deform into exotic configurations. In this talk, we explore the link between the regularity of the prestrain and the minimum energy. An upper bound on the energy can be derived in terms of the regularity of the prestrain; a lower bound follows from known irregularities. The bounds match up to corrections that seem to require finer details of the prestrain. This is joint work with Ian Tobasco.

Valerio Pagliari, Technische Universität Wien

Convergence of nonlocal geometric flows to anisotropic mean curvature motion

We consider the Cauchy problem for a class of motions by nonlocal curvature, whose wellposedness may be established in the framework of viscosity solutions. We show that a suitable rescaling of the nonlocal curvatures induces a localisation effect, that is, as the scaling parameter tends to 0, we retrieve an anisotropic mean curvature functional. As a consequence, by means of the theory of geometric barriers by De Giorgi, we are able to prove that the solutions to the rescaled nonlocal motions locally uniformly converge to the solution of the local flow. The talk is based on joint work with A. Cesaroni (Padova).

Claudia Raithel, Technische Universität Wien

Quantitative homogenization for the case of an interface between two heterogeneous media

In this talk we are interested in quantitative homogenization results for linear elliptic equations in the situation of a straight interface between two het- erogenous media. The setting that we consider allows for the surrounding media to be periodic, almost-periodic, or stationary and ergodic. The presence of a straight interface, which, e.g., might occur when studying crystals, makes the environment non-stationary and, therefore, outside of the framework usually handled by classsical methods in stochastic homogenization. The main result that we obtain is a quantification of the sublinearity properties of a homogenization corrector that we construct to be adapted to the interface. This quantification is optimal up to a logarithmic loss and allows us to prove almost-optimal convergence rates.

This talk is based on joint work with M. Josien.

Alberto Roncoroni, University of Florence

Rigidity results for quasilinear elliptic equations with Neumann boundary conditions

In this talk, we present some rigidity (non-existence and classification) results for (possibly anisotropic) quasi-linear elliptic PDEs in bounded convex domain of $\mathbb{R}^n$. The model equation is p-Laplace type and includes the critical p-Laplace type equations arising from Sobolev type inequalities. Under a suitable condition on the nonlinearity (which is optimal) and assuming that a Neumann (or more in general Robin) boundary condition is imposed at the boundary $\partial\Omega$, we show that the constants are the only positive bounded weak solutions. A relevant con- sequence of our results is that we can extend to weak solutions the celebrated results obtained for stable solutions by Casten, Holland (1978) and by Matano (1979).

This is based on a joint work with G. Ciraolo and R. Corso.

Marius Zeinhofer, Albert-Ludwigs-Universität Freiburg

Analysis of Scaffold Mediated Bone Growth

The regeneration and restoration of skeletal functions of critical-sized bone defects are very challenging. Presently, the treatment strategy using a porous, bio-resorbable scaffold as a temporary support structure is being investigated. We present a three dimensional, time dependent PDE/ODE model for bone regeneration in the presence of porous scaffolds to bridge critical size bone defects. Our approach uses homogenized quantities, thus drastically reducing computational cost and allows for an optimization of the scaffold’s density distribution. We discuss well-posedness of the model, existence results of the associated optimization problem and present numerical simulations.